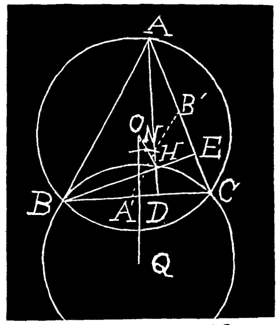
GEOMETRY.

338. Proposed by C. N. SCHMALL, 239 East 7th Street, New York.

Given the base and vertical angle of a triangle, find the locus of the center of its nine-point circle.

I. Solution by J. W. CLAWSON, Ursinus College, Collegeville, Pa.

(1) Take an angle BAC equal to the given vertical angle and from any point B in one arm of it describe an arc with BC equal to the given base as radius cutting the other arm in C. Draw a circle through A, B, C. Then BC being fixed, the circle ABC is the locus of the vertex A.



(2) Find the center O of this circle, draw OA' the perpendicular bisector of BC, and OB' the perpendicular bisector of CA. Drop AD perpendicular to BC, and BE perpendicular to CA, intersecting AD in H.

The nine-point circle passes through A', D and B', E. Therefore the lines bisecting $A'D$ and $B'E$ at right angles intersect in N, the center of the nine-point circle. But evidently each of these lines bisects OH. Therefore N bisects OH.

(3) Join $A'B'$. The triangles AHB and $OA'B'$ are similar, having their sides mutually parallel. Also $AB=2A'B'$. So $AH=2OA'$, and is therefore constant.

(4) Since AH is constant in length, and as A moves AH moves parallel to itself, the locus of H is a circle of radius equal to OA' and whose center, Q, lies on OA' produced so that $OQ=AH=2OA'$. Draw this circle.

(5) Since N bisects OH, O is a fixed point, and the locus of H is a circle, the locus of N is a circle, whose center bisects OQ, and is therefore the point A', and whose radius is half the radius of the original circle.

Also solved by J. M. Meyer, S. J,, Francis Rust, V. M. Spunar, G. B. M. Zerr, and J. Scheffer.

CALCULUS.

265. Proposed by V. M. SPUNAR, M. and E. E., East Pittsburg, Pa.

Find two curves which possess the property that the tangents TP and TQ to the inner one always make equal angles with the tangent TT' to the outer.

II. Solution by F. H. SAFFORD, Ph. D., University of Pennsylvania.

Let $y=\phi(x)$ be the equation of the inner curve, and let the coordinates of P and Q, points on this curve, be, respectively, (x_1, y_1), (x_2, y_2). Assuming that the outer curve exists, let its equation be $y=f(x)$, and let $f'(x)=\tan\alpha$.

The two tangent lines from P and Q, respectively, are, if $\phi'(x)=\tan\theta$,